Physics 151 Class Exercise: Simple Harmonic Motion

1. A 1.4 kg mass is attached to a large horizontal spring on the top of a table. The mass is pulled 12 cm from the equilibrium position and released. It then undergoes simple harmonic motion making 2.2 oscillations each second. Determine:

(a) the equation of motion

$$x(t) = x_0 \cos\left(2\pi \frac{t}{T}\right) = (0.12m)\cos(13.8t)$$

b) the spring constant

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$f^2 = \frac{1}{4\pi^2}\frac{k}{m}$$

$$k = 4\pi^2 f^2 m = 4\pi^2 (2.2Hz)^2 (1.4kg) = 267.5\frac{N}{m} = 267\frac{N}{m}$$

c) the total energy

$$E = \frac{1}{2}kx_0^2 = \frac{1}{2}\left(267.5\frac{N}{m}\right)(0.12m)^2 = 1.926J = 1.93J$$

d) the maximum acceleration of the mass (and indicate where this occurs)

$$a = \frac{kx_0}{m} = \frac{\left(267.5\frac{N}{m}\right)(-0.12m)}{(1.4kg)} = 22.9\frac{m}{s^2}$$ the negative amplitude position

e) the maximum velocity of the mass (and indicate where this occurs)

$$v = x_0\sqrt{\frac{k}{m}} = (0.12m)\sqrt{\frac{\left(267.5\frac{N}{m}\right)}{(1.4kg)}} = 1.66\frac{m}{s}$$ at the equilibrium position

f) the acceleration of the mass when it is 7 cm from the equilibrium position

$$a = -\frac{kx}{m} = -\frac{\left(267.5\frac{N}{m}\right)(0.07m)}{(1.4kg)} = -13.4\frac{m}{s^2}$$

g) the velocity of mass when it is 7 cm from the equilibrium position

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$v = \sqrt{\frac{2E - kx^2}{m}} = \sqrt{\frac{2(1.93J) - \left(267.5\frac{N}{m}\right)(0.07m)^2}{(1.4kg)}} = 1.35\frac{m}{s}$$